Journal of Applied Mechanics and Technical Physics, Vol. 35, No. 2, 1994

DESIGN AND TESTING OF HIGH-PRESSURE COVERS

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UDC 620.178.7

Pressure-vessel covers mainly work under quasistatic pressure, although there are cases of pulsed loading, such as in protective equipment and systems with explosive or other energy release. Here we consider only calculating the optimum shape for a cover for a circular hole in quasistatic loading by a uniform pressure.

The equal-strength principle [1] implies that the maximal equivalent stresses σ_i attain the yield point σ_s at each point in a deformable linearly elastic body. For example, this can be applied to optimize a clamped circular plate under uniform pressure, where the mass is reduced by 20% by profiling the thickness [1]. The calculation scheme for the cover is that of a thin shell of rotation, in which we incorporate only the membrane components of the stresses and neglect edge effects.

We consider technologically simple cover forms.

Spherical Segment with Radius *a* at the Base (*a* is the Radius of the Hole to be Covered) and with Unknown Sphericity Radius R. Here equal strength is provided by constant shell thickness δ. One determines the necessary mass m from the Mises yield criterion on the basis that the principal stresses are $\sigma_1 = \sigma_2 = pR/2\delta = \sigma_s$, which gives for a pressure p that

$$m = \frac{\pi p \rho a^3}{\sigma_s}\frac{1}{\xi^3}\left(1 - \sqrt{1-\xi^2}\right),$$

in which $\xi = a/R$ $(0 < \xi < 1)$, and ρ is the density. The minimum mass is attained for $\xi = 0.8660$, i.e., with $R_{opt} = 1.155a$:

$$\min(m) = \frac{\pi p \rho a^3}{\sigma_s}0.7698.$$

The height of the cover is $h_{opt} = 0.5775a$.

Conical Cover with Semivertex Angle φ and Unknown Dependence of Thickness δ on Current Radius r $(0 \le r \le a)$. The following are the meridional stressed σ_φ and annular ones σ_θ [2]:

$$\sigma_\varphi = \frac{pr}{2\delta(r)\cos\varphi},\ \sigma_\theta = \frac{pr}{\delta(r)\cos\varphi} = 2\sigma_\varphi.$$

We derive $\delta(r)$ providing yield throughout the volume of the material in the conical shell:

$$\sigma_s = \sigma_i = \frac{1}{\sqrt{2}}\sqrt{(\sigma_\varphi - \sigma_\theta)^2 + \sigma_\varphi^2 + \sigma_\theta^2} = \frac{\sqrt{3}pr}{2\delta(r)}f_1(\varphi)\ \left(f_1(\varphi) = \frac{1}{\cos\varphi}\right).$$

Then $\delta(r) = \sqrt{3}pr/2\sigma_s f_1{}^*(\varphi)$ = const r for a given φ, i.e., δ is proportional to r*. Integration gives the mass as

$$m = \frac{1}{\sqrt{3}}\frac{\pi p \rho a^3}{\sigma_s}f_2(\varphi)\ \left(f_2(\varphi) = \frac{1}{\sin\varphi\cos\varphi}\right).$$

*Although $\delta \to 0$ for $r \to 0$ is physically unrealizable, it is always possible to produce the required thickening in the region of the pole with only a minor effect on the total mass.

Arzamas. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 2, pp. 163-168, March-April, 1994. Original article submitted October 12, 1992; revision submitted April 24, 1993.

We minimize f_2 with respect to φ to get $\min(f_2) = 2.0$ for $\varphi_{opt} = 45°$. The optimal mass is $\min(m) = 1.1547\ \pi p\rho a^3/\sigma_s$, which is about 50% greater than the mass of an optimal cover as a spherical segment. The height of the cover is $h_{opt} = a$, which is 73.2% larger than in the previous case. The necessary thickness is

$$\delta(r) = \frac{pr}{\sigma_s}1.2247.$$

These shapes (sphere and cone) are strictly free from moments when the corresponding edge conditions are met because: 1) the moment-free condition [2] $v = -w'$ is met for a sphere, where $v = 0$ is the displacement along the tangent to the meridian and $w' = 0$ is the derivative of the displacement w along the normal to the shell with respect to the meridian angle, and 2) in the case of a cone, the radius of curvature of the section for the median surface along the meridian (r_1 in [2] symbols) is equal to infinity, and from (3) and (4) in [2] we get that the bending moments $M_\varphi = M_\theta = 0$, so these solutions are actually optimal (within the accuracy of the membrane approximation).

Interest attaches to whether there are other shapes for equal strength covers* (Fig. 1) with less minimal mass. We consider deriving the shape r(x) of the cover that minimizes the mass for given p = const and a while satisfying the equal-strength condition. Then φ is variable and $\cos\varphi = \frac{1}{\sqrt{1 + r_x^2}} = \frac{1}{f_3(r_x)}\ \left(r_x = \frac{dr}{dx}\right)$ and similarly we get

$$\sigma_\varphi = \frac{pr}{2\delta(r)}f_3(r_x),\ \sigma_\theta = 2\sigma_\varphi\left[1 + \frac{rr_{xx}}{2(1 + r_x^2)}\right]\ \left(r_{xx} = \frac{dr_x}{dx}\right).$$

The yield condition gives $\sigma_s = \frac{pr}{\sqrt{2}\delta(r)}f_4(r, r_x, r_{xx})$, whence $\delta(r) = \frac{prf_4(r, r_x, r_{xx})}{\sqrt{2}\sigma_s}$. The form of $f_4(r, r_x, r_{xx})$ dependent on the yield of criterion. A mass element is

$$dm = 2\pi r\delta\rho ds = \frac{\sqrt{2}\pi p\rho r^2}{\sigma_s}f_4(r, r_x, r_{xx})\sqrt{1 + r_x^2}dx.$$

The total shell mass is

$$m = \sqrt{2}\pi\frac{p\rho}{\sigma_s}\int_0^h r^2 f_4(r, r_x, r_{xx})\sqrt{1 + r_x^2}dx.$$

Transformation gives (with the Mises criterion)

$$m = \pi\frac{p\rho}{\sigma_s}\int_0^h r^2\sqrt{3(1 + r_x^2)^2 + 3rr_{xx}(1 + r_x^2) + r^2r_{xx}^2}dx = \frac{\pi p\rho}{\sigma_s}J. \tag{1}$$

To minimize m, one has to solve the variational problem for the integral J having variable bound h in relation to the function r(x). That function should be smooth, positive-definite, and satisfy the boundary conditions r(0) = 0 at x = 0 and r(h) = a at x = h = βa.

For estimation purposes, we neglect the additional constraints on r(x) needed to provide a strict moment-free shape. This gives an estimate for the minimum mass with a safety margin, since one considers a wider class of shapes than for that constraint. If the result for r(x) that minimizes m is not strictly free from moments, one needs to increase m to compensate for the incomplete homogeneity in the state of strain. The solution is derived numerically by minimizing the functional on approximating r(x) by a polynomial:

$$r(x) = \frac{1}{\beta}x + \sum_{i=1}^{N}\alpha_i(x^i - \beta^i a^i)x$$

(first form) or a power law $r(x) = a\left(\frac{x}{h}\right)^\gamma$ second form), which satisfy the boundary conditions.

We have determined the parameters $\beta = h/a$, α_1, $\alpha_2 (N = 2)$ and γ that minimize J in (1). The following local minima have been found

$$\beta = 0.6465,\ \alpha_1 = -3.4073,\ \alpha_2 = 1.3783,\ J = 0.7736a^3; \tag{a}$$

$$\beta = 0.4904,\ \gamma = 0.4197,\ J = 0.8547a^3. \tag{b}$$

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*A solution has been obtained [3] for the shape of an equal-resistance dome under its own weight.

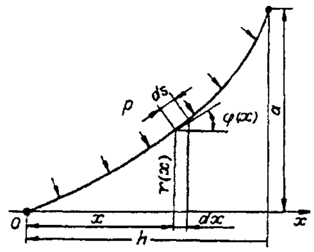

Fig. 1

Solution a is better than the solution for the optimal sphere by 1%, while solution b is better by 11% as regards optimal mass relative to a sphere and with a relative height β less by 15%. Increasing the number N of terms summed in the polynomial for r(x) from 2 to 3 allows one to reduce J from $0.7736a^3$ to $0.7588a^3$, which about 1.4% less than for a sphere. This is attained with $\beta = 0.6116$, $\alpha_1 = -7.7565$, $\alpha_2 = 8.2749$, $\alpha_3 = -3.7832$. Although one can increase the number of series terms, there is no guarantee of finding the global minimum [1], and one can also use other approximations for r(x), but the estimate does show that one should not expect any substantial advantage from varying the form by comparison with the optimal spherical one. Figures 2 and 3 show the profiles and thicknesses, where we show the $\bar{r}(\bar{x})$ dependence in $\bar{r} = r/a$, $\bar{x} = x/a$ coordinates together with $\bar{\delta}(\bar{x})$* ($\bar{\delta} = 2\sigma_s\delta/p$).

These estimates and design limitations led us to adopt a cone with semivertex angle 52° for realization of a strong cover, which passes without a kink on the outer surface into a spherical segment of the same thickness (Fig. 4). That shape is preferable for dynamic loading on close explosion of a charge. The spherical segment is linked to the cone along the tangent to the outer surface without thickness change, and it is 41% stronger than the cone. The line of contact between the sphere and the cone has a kink on the inner surface. The step in the curvature and the kink in the generator do not allow one to attain the moment free state in the linkage zone. The moment-free state is also violated near the supporting rings. This requires a two-dimensional analysis of edge effects in the state of stress and strain and the carrying capacity of the actual structure. We used the SINTEZ program, which employs the finite-element method for an axisymmetric body subject to elastic and plastic strains [4]. We found that yield sets in at the point $\sigma_{i\ max}$ (Fig. 4) in the real structure at a load of 77.2% of the limit given by the membrane-scheme estimates. The yield begins throughout the thickness of the cover at a pressure about 148% of the limit estimated for the cone. These results are independent of the bearing conditions on surfaces A and B (we considered hinge support for the finite-element nodes on A and B, on the outer part of A, and on B, and also hinge support only for A). The result is evidently affected by the reinforcing effect from the spherical segment.

We checked the results by hydraulic test on a cover made of steel having the following parameters: yield point σ_s = 1.14-1.2 GPa, and relative elongation σ_b = 1.32-1.34 GPa (from tests on four reference specimens after a heat treatment cycle). The two-dimensional calculation indicates that the start of yield corresponds to p_s = 65.7 MPa (the estimates for the cone give p_s = 85.1 MPa). These calculations imply that a cover with this shape has a carrying capacity more than 14.5 times greater than that of an optimized cover with the same mass made of the same material and having the same diameter as a circular gripped plate with profiled thickness [1]. Strain-gauge measurements at three points on the inner surface confirmed the two-dimensional elastic-strain calculations. The following loading stages were implemented:

1) 0-50 MPa, reset to zero, lengths of stages 3 ± 1 min;

2) 0-110 MPa, reset to zero;

3) 0-240 MPa, failure in sealing and sensor detachment;

4) 0-190 MPa, that pressure could not be exceeded.

Examination after the fourth stage showed a dent of diameter about 30 mm and depth about 6 mm in the region of the $\sigma_{i\ max}$ point (Fig. 4).

The strain gauge measurements showed that there was no residual strain in stage 1, while after stage 2, some of the sensors recorded residual strains of ≤ 0.06%. After stage 3, although the pressure attained 240 MPa, there was no stability loss in the initial stage during yield (to judge from the external appearance of the cover). Although the cover material went over to the yield state, the cover could withstand a pressure 20-50% higher than the calculated value for yield in the entire volume without loss of stability. This increase in carrying capacity could not be explained completely from the available data

*For a cover in the second form (power-law form), $\delta \to 0$, which is not physically realizable, as in the case of the cone with $\delta \to 0$.

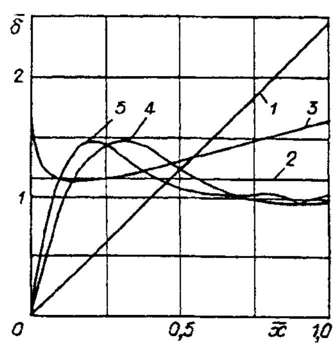

Fig. 2

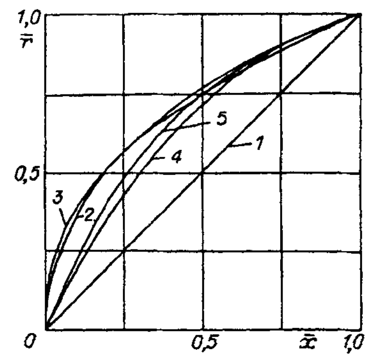

Fig. 3

Fig. 2. Cover thickness profiles: 1) cone; 2) sphere; 3) solution b; 4) solution a, N = 2; 5) solution a, N = 3.

Fig. 3. Cover radius profiles: 1) cone; 2) sphere; 3) solution b; 4) solution a, N = 2; 5) solution a, N = 3.

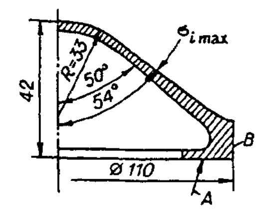

Fig. 4

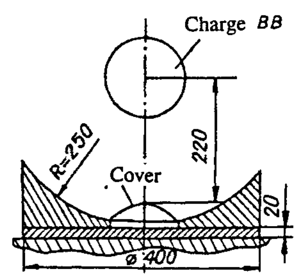

Fig. 5

on the $\sigma-\varepsilon$ stretching diagram and on the basis of the shell thickening in biaxial compression. Qualitatively, the effect may be due to work hardening under compound loading (biaxial elastoplastic compression with unequal stress components and with several loading-unloading cycles) [5]. We also performed two tests on this cover with explosive loading as in Fig. 5. An alloy of trotyl with hexogen in a 50:50 proportion was used, which was detonated from the center. The mass was 2.2 kg.

In the first test, the cover was installed on a flat circular steel plate lying on a basement plate, while in the second we added a component to stimulate a spherical cavity with radius 250 mm in the region of the cover with diameter 400 mm. The covers after loading had not lost stability, and in the first case the maximal residual annular compressive strain was 0.25%, while the maximum dynamic value was 0.9%. In the second test, we obtained strains correspondingly of 0.8 and 1.4%. This shows that there is comparatively high resistance to stability loss in plastic strain on dynamic loading. The shock-wave reflection pressure at the pole of the cover was estimated by calculation [6] as about 380 MPa, which was more than twice the limiting quasistatic pressure.

The convexity was facing the pressure, i.e., the material worked in biaxial compression, which is preferable to the converse from the viewpoint of raising the strength and from the failure-mechanics aspect, since initial defects of detachment crack type cannot grow in that case. Additional research is required on the effects of initial defects on the failure from the advance of shear cracks during plastic flow under biaxial compression and on the stability of that process.

Estimates have been made on the shape of a cover near-optimal in mass to cover a circular hole in a pressure vessel. Hydraulic tests and explosive loading show satisfactory agreement between theory and experiment for the cover shape and realized and also that the carrying capacity is greatly increased with the proposed design by comparison with a flat circular plate having optimized thickness.

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